CallCulator Handbook

Advanced version

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Part I
Foundations

1 Welcome to CallCulator

CallCulator is an interactive web-based tool for quantitative options analysis. It enables users to move beyond zero-dimensional metrics (price targets) and instead express their market beliefs through probability distributions, which are then combined with risk tolerance profiles to find mathematically optimal investment strategies.

The platform offers three main modes, accessible via the tabs at the top of the page:

1. **Basic Calculation** – Manually define a custom option spread and compute its full performance profile, Black-Scholes time-decay surface, and Monte-Carlo CAGR statistics.
2. **Probability & Risk** – Supply a probability distribution and risk tolerance to let the backend search engine find the globally optimal strategy across hundreds of millions of option combinations.
3. **Max Min** – Find strategies that maximize the worst-case outcome (minimax optimization), either for a single stock or across a multi-stock portfolio.

2 Basics of Options Trading

2.1 What is an Option?

An **option** is a financial derivative that grants the holder the right—but not the obligation—to buy or sell an underlying asset at a pre-agreed price (the **strike price** K) on or before a specified date (the **expiration date** T).

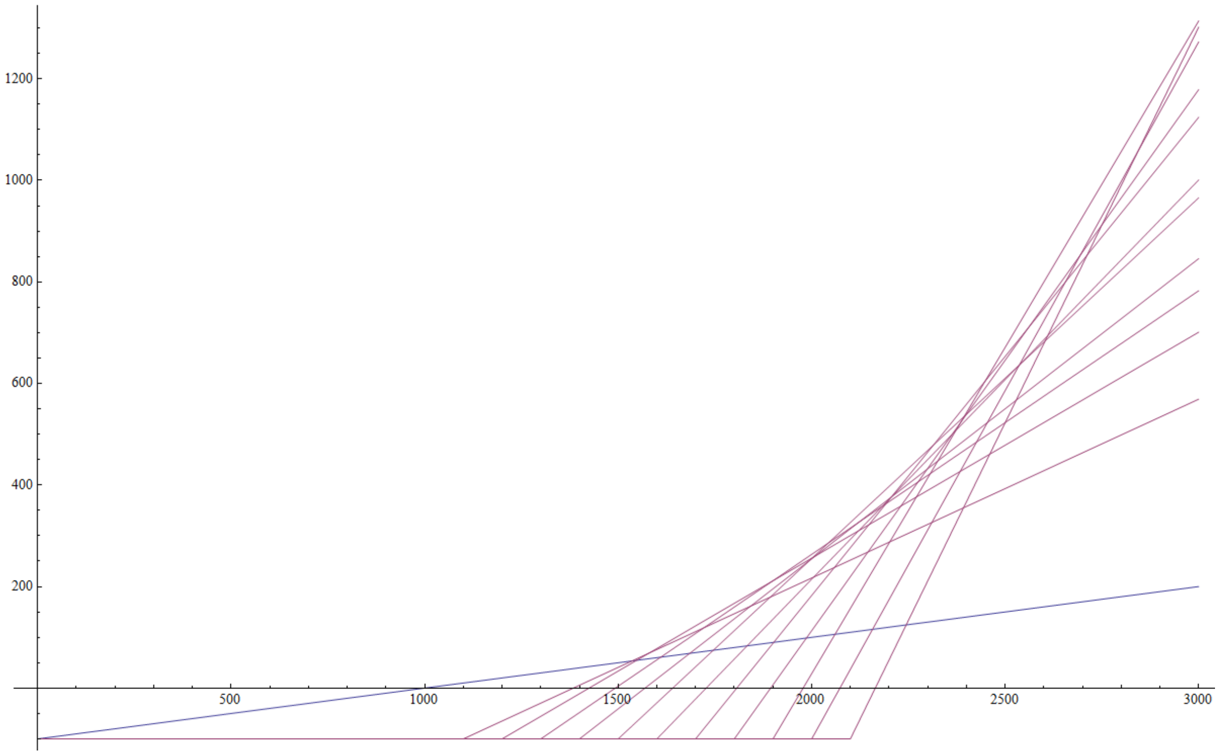
- **Call Option** $C(K, T)$: Right to *buy* at K. Payoff at expiration: $\max(S_T - K, 0)$ where S_T is the stock price at time T.
- **Put Option** $P(K, T)$: Right to *sell* at K. Payoff at expiration: $\max(K - S_T, 0)$.

The **premium** p is the market price of the option. In percentage terms, the profit/loss of a call option is:

$$g_c(S_T) = \max\Big(-1,\ \frac{S_T - K - p}{p}\Big) \tag{1}$$

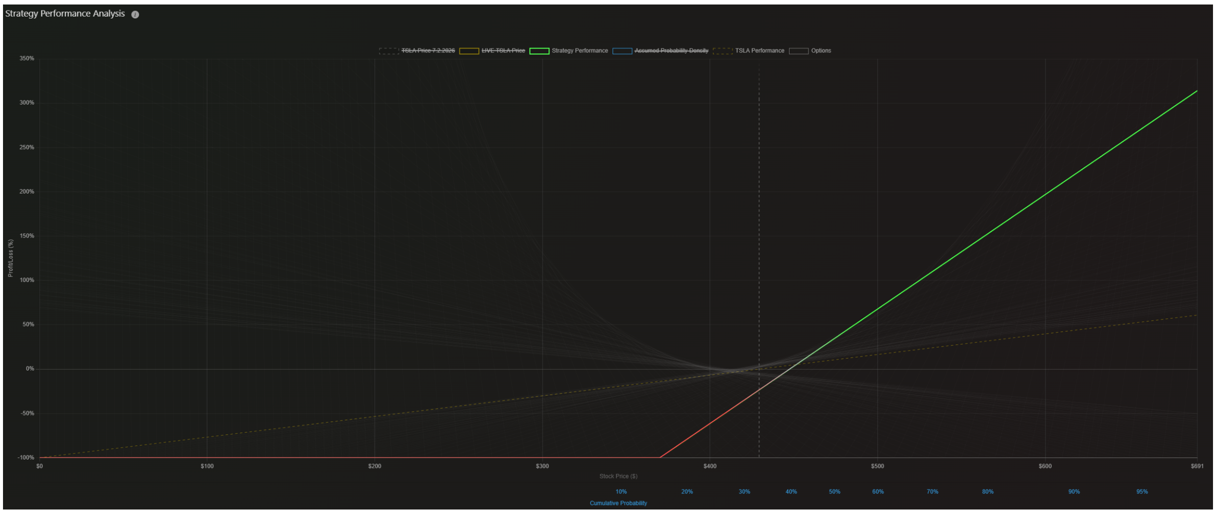
2.2 Visualizing Option Performance

The graph below shows the percentage gain or loss of different call options (red lines) compared to simply owning the stock (blue line):



Red: percentage gain of various call options. Blue: the underlying stock.

Notice how the red lines are much steeper than the blue line. A 10% rise in the stock price might translate to a 50% or even 200% rise in a call option's value. But if the stock doesn't reach the strike price, the option goes to zero—a 100% loss. This is the leverage effect in action.



A single call option's performance at expiration.

2.3 American vs. European Options

While you can trade (buy/sell) any option on the market at any time, there is a distinction in when you can **exercise** the right to buy or sell the underlying stock:

- **American-Style:** You can exercise the option at any point up to and including the expiration date. Most standard equity options are American-style.
- **European-Style:** You can only exercise the option on the expiration date itself. Index options are often European-style.

For most traders, this distinction is technical because options are rarely exercised early; they are usually sold to close the position.

2.4 What is a Spread?

A **spread** is a convex combination of financial products:

$$\sum_i w_i\, g_i \quad \text{where} \quad \sum_i w_i = 1,\ w_i \in [0,1] \tag{2}$$

Spreads allow you to engineer a specific risk/reward profile. CallCulator supports multi-leg strategies including **short (write) positions**, enabling bull spreads, bear spreads, iron condors, and custom combinations. The search algorithms don't know any difference between those traditionally grouped spreads into names, it is completely arbitrary.

Part II
Core Philosophy: Why Probability Distributions?

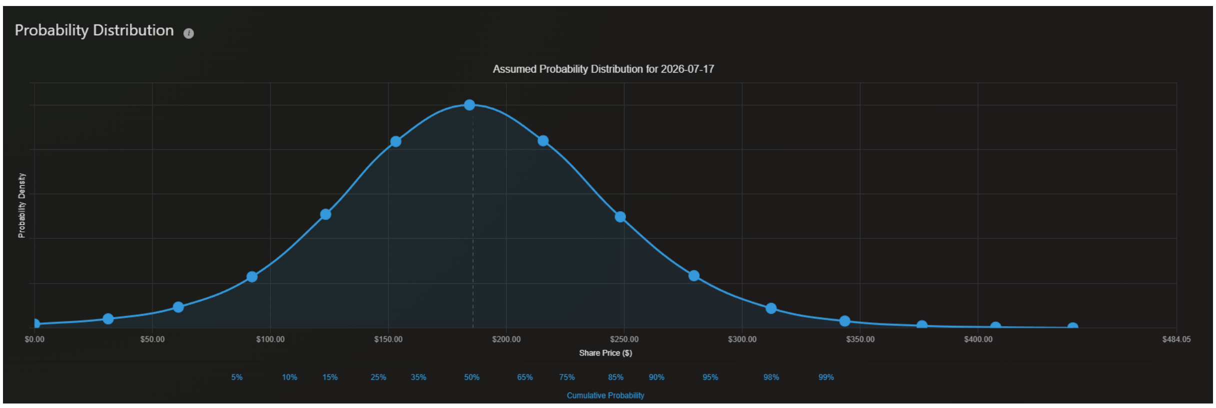
3 The Problem with Price Targets

Conventional analysis relies on a "price target"—a single number X representing where you think a stock will be by some date. This is a zero-dimensional metric that collapses all uncertainty into a point estimate.

The problem: two investors may both have a target of \$150, yet one might be very confident (narrow bell curve around \$150) while the other sees wild uncertainty (flat distribution from \$80 to \$220). A price target cannot distinguish these beliefs, leading to fundamentally different appropriate strategies.

4 Our Approach: Probability Density Functions

CallCulator replaces the price target with a **Probability Density Function (PDF)** $\mu(x)$ for the stock price at a future date. This captures the full shape of uncertainty:

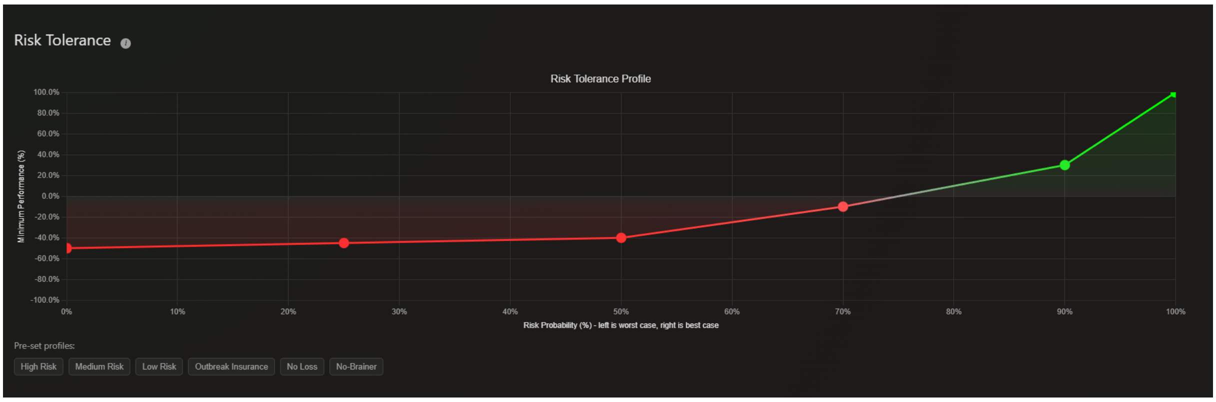


A probability distribution supplied by the user: the peak at $\sim$ \$180 indicates the most likely price.

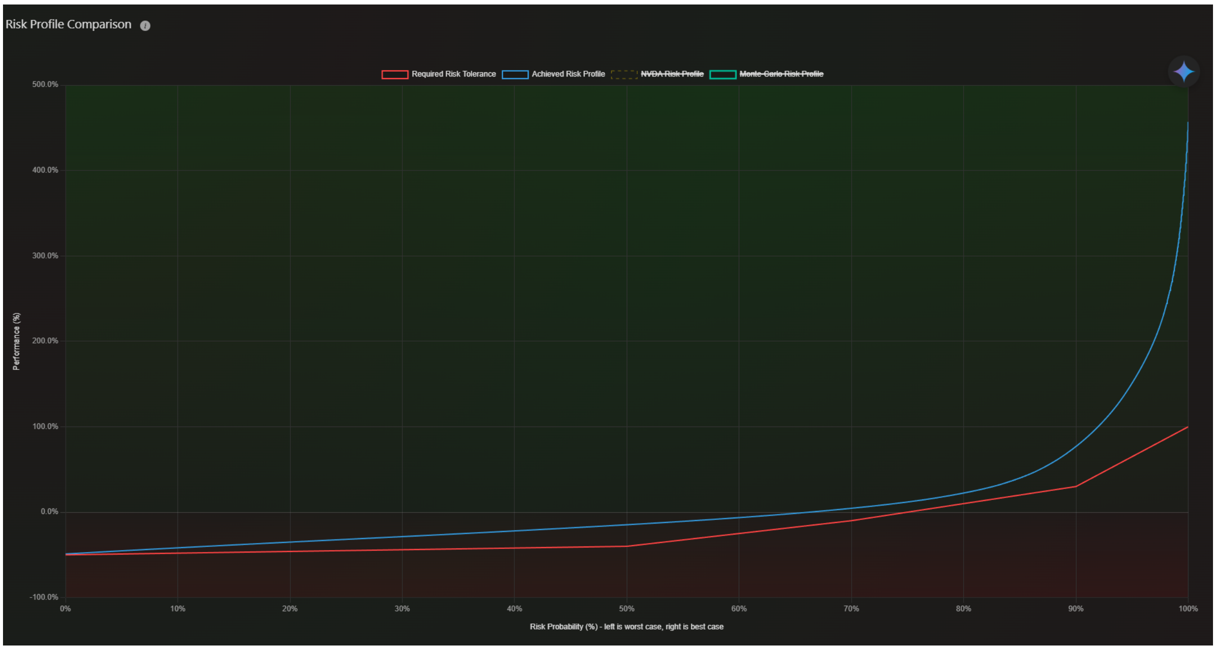
Given $\mu(x)$ and a set of available options $\{g_i\}$, the **expected return** of a strategy is:

$$\mathbb{E}_\mu\Big[\sum_i w_i\, g_i\Big] = \sum_i w_i \int_{\mathbb{R}} g_i(x)\, \mu(x)\, dx \tag{3}$$

Combined with a **Risk Tolerance Profile**, we can find the strategy that maximizes expected return subject to the constraint that its risk profile stays within the user's tolerance at every cumulative probability level.



The user's risk tolerance profile $\hat{r}_{tol}(\lambda)$: a continuous boundary from worst case (left) to best case (right).

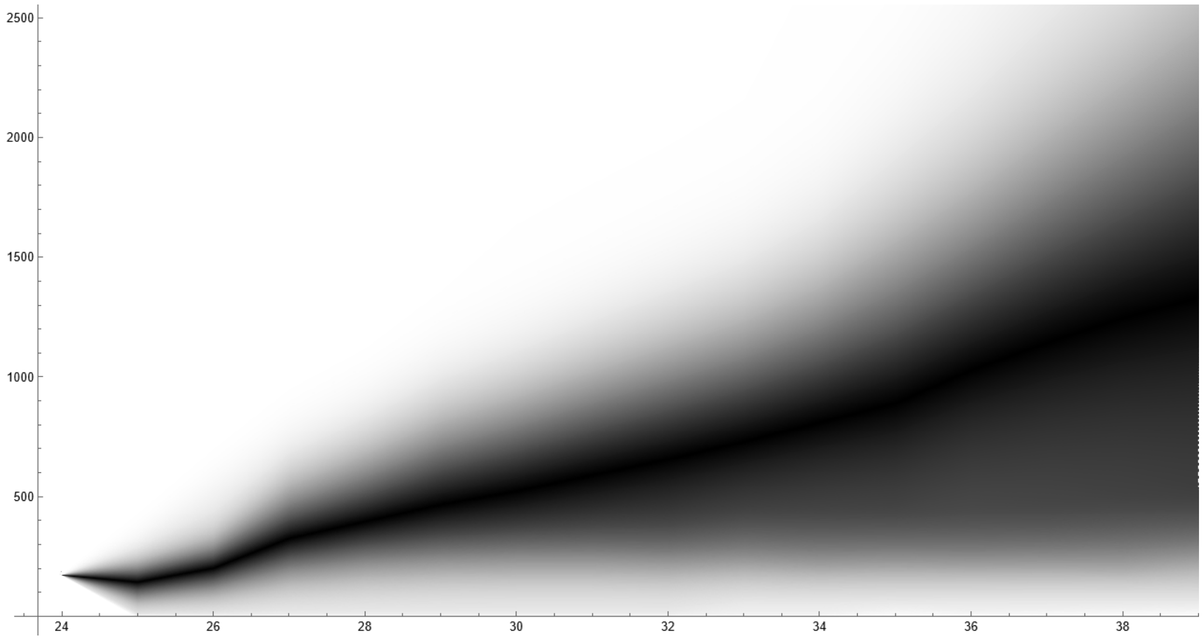


The found strategy's risk profile $\hat{r}_c(\lambda)$ (blue) satisfying $\hat{r}_c(\lambda) \geq \hat{r}_{tol}(\lambda)\ \forall \lambda \in [0, 1]$

5 Probability Distributions Over Time (In Development)

For dynamic strategies, we model beliefs as a family of distributions $\{\mu_i(x)\}_{i\in[n]}$ at times $\{d_i\}$, then interpolate continuously using linear transport:

$$\nu(x, d) = \frac{d_{i+1} - d}{d_{i+1} - d_i}\, \mu_i(x) + \frac{d - d_i}{d_{i+1} - d_i}\, \mu_{i+1}(x), \quad d \in [d_i, d_{i+1}] \tag{4}$$



Grayscale heatmap of a probability distribution evolving over time.

Part III
Using CallCulator

6 Basic Calculation Mode

6.1 Workflow

1. **Enter a Ticker:** Type a stock symbol (e.g., `TSLA`) and press Enter. The system fetches the current price, company name, FIGI code, and full options chain via the backend Data Manager.
2. **Select an Expiration Date:** Choose from the list of available expirations.
3. **Build Your Strategy:** Add legs (Buy Call, Buy Put, Sell Call, Sell Put) with strike prices and weights (percentages representing capital allocation, e.g., 60% + 40%).
4. **Compute:** The backend `BasicCalcs` Lambda calculates the full analysis.

6.2 Results Displayed

- **Strategy Performance Chart:** A graph showing your percentage gain/loss (y-axis) at every possible stock price (x-axis) at expiration.
- **Black-Scholes Table:** A grid of estimated option values across time (columns) and stock price (rows), using the Black-Scholes PDE solution. Color-coded: green = profit, red = loss.

  The table axes are:

  - **X-axis (Time):** Time progresses from left (today) to right (expiration date).
  - **Y-axis (Price):** Possible share prices of the underlying stock.

  The cell at coordinate (x, y) shows the estimated P/L of your strategy at that specific future time and stock price. The rightmost column (at expiration) corresponds exactly to the values in the Strategy Performance Chart above it (just rotated 90 degrees). Values at expiration are certain (intrinsic value), while values earlier in time are theoretical estimates based on the Black-Scholes model.

- **Monte-Carlo CAGR Statistics:** Simulated compound annual growth rate assuming repeated investment with the given probability distribution.
- **Action Buttons:** Share (generates public link), Track, Invest (in development).

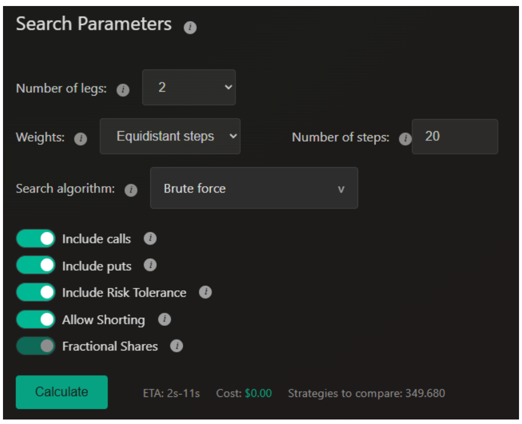
Note on Black-Scholes realism: The standard Black-Scholes model assumes constant implied volatility. In reality, if a stock jumps from \$150 to \$600, the implied volatility would spike dramatically, making the actual option value far higher than the static model predicts. This motivates the development of *Dynamic Black-Scholes* models.

7 Probability & Risk Mode

This is CallCulator's flagship mode. Instead of manually defining a strategy, you define your *beliefs* and let the optimization engine find the best strategy.

7.1 Workflow

1. **Enter a Ticker:** Same as Basic mode.
2. **Select an Expiration Date:** The system pre-fetches all implied PDFs in a single backend call (`/impliedPDFAll`).
3. **Define Your Probability Distribution:** Adjust the probability curve $\mu(x)$ to match your belief about where the stock will be at expiration (see figure below).
4. **Set Risk Tolerance:** Define a continuous tolerance function $\hat{r}_{tol}(\lambda)$ mapping each cumulative probability λ to the minimum acceptable return at that level (e.g., "in the worst 40% of outcomes, I require at least -5%").
5. **Configure Search Parameters:** Choose the number of **legs** (option positions per strategy), the number of **steps** (granularity of the strike/weight grid), and the search algorithm. Currently a **Brute Force** scan is used; smarter algorithms (Quasi-Monte Carlo / Sobol seeding, adaptive hill-climbing) are in development. Complexity grows rapidly with legs and steps, affecting computation time and cost.



Search parameter configuration panel.

6. **Launch Search:** The backend Search Manager distributes the optimization across multiple Lambda worker nodes, each evaluating hundreds of millions of option combinations in parallel (up to 625 threads per node, ∼80 calculations per thread).

7.2 The Optimization Engine

The backend considers all 2- and 3-leg option spreads from the available chain. The optimization problem is formally defined as:

$$\max_{s \in \mathcal{S}} \mathbb{E}_P[g_s] = \max_{s \in \mathcal{S}} \int_0^\infty g_s(x) P(x)\, dx \tag{5}$$

Subject to the risk constraint:

$$\forall q \in [0,1] : R_s^{-1}(q) \geq T(q) \tag{6}$$

where $T(q)$ is the user's risk tolerance profile (minimum return at quantile q), and $R_s(y)$ is the cumulative distribution function of the strategy's return:

$$R_s(y) = \int_{\{x | g_s(x) \leq y\}} P(x)\, dx \tag{7}$$

Expanding the strategy s as a weighted sum of options $o_i \in \mathcal{O}$:

$$\max_{\substack{o_i \in \mathcal{O} \\ w_i \in \mathbb{R} \\ \sum w_i = 1}} \int_0^\infty \left(\sum_i w_i \cdot g_{o_i}(x) \right) P(x)\, dx \tag{8}$$

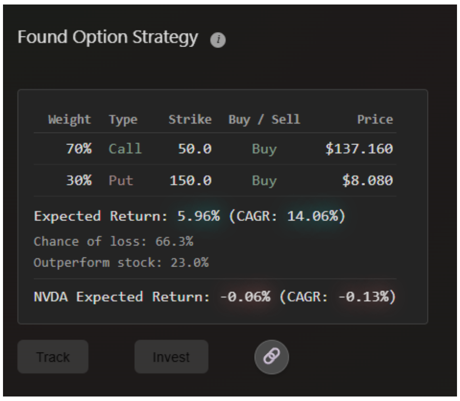
For each candidate:

1. Compute the performance function $g(S_T)$ using spline interpolation.
2. Compute the expected return: $\mathbb{E}_\mu[g] = \int g(x)\, \mu(x)\, dx$.
3. Compute the risk profile and verify it satisfies the risk tolerance constraint.
4. If valid, compare against the current best strategy.

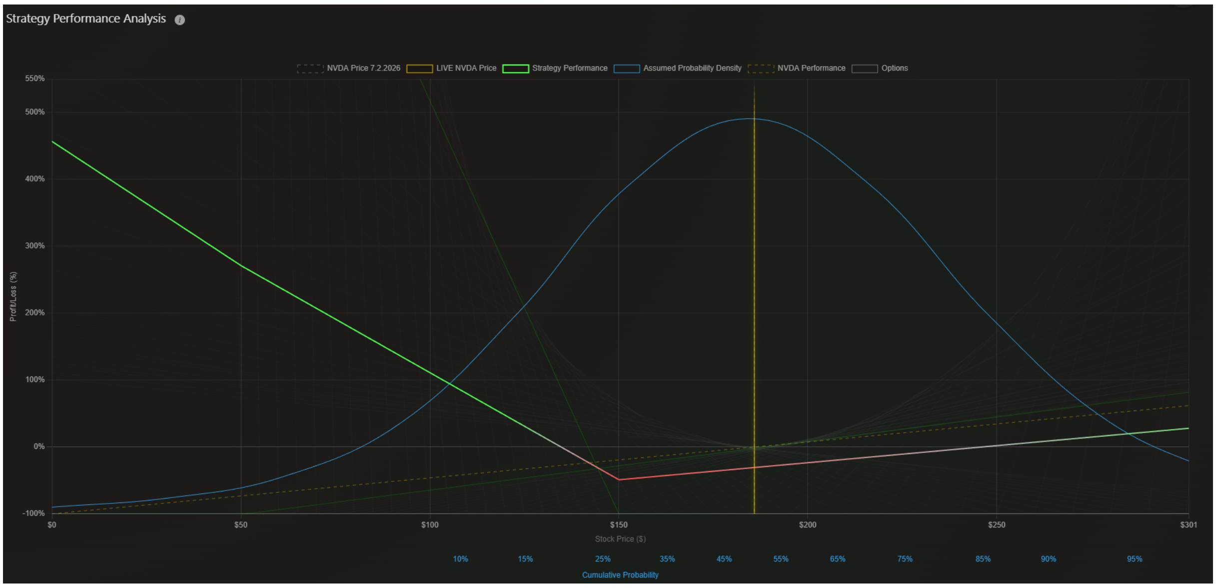
Weights represent **capital allocation** (percentage of portfolio). The backend derives option quantities as weight/option_price.

7.3 Results Displayed

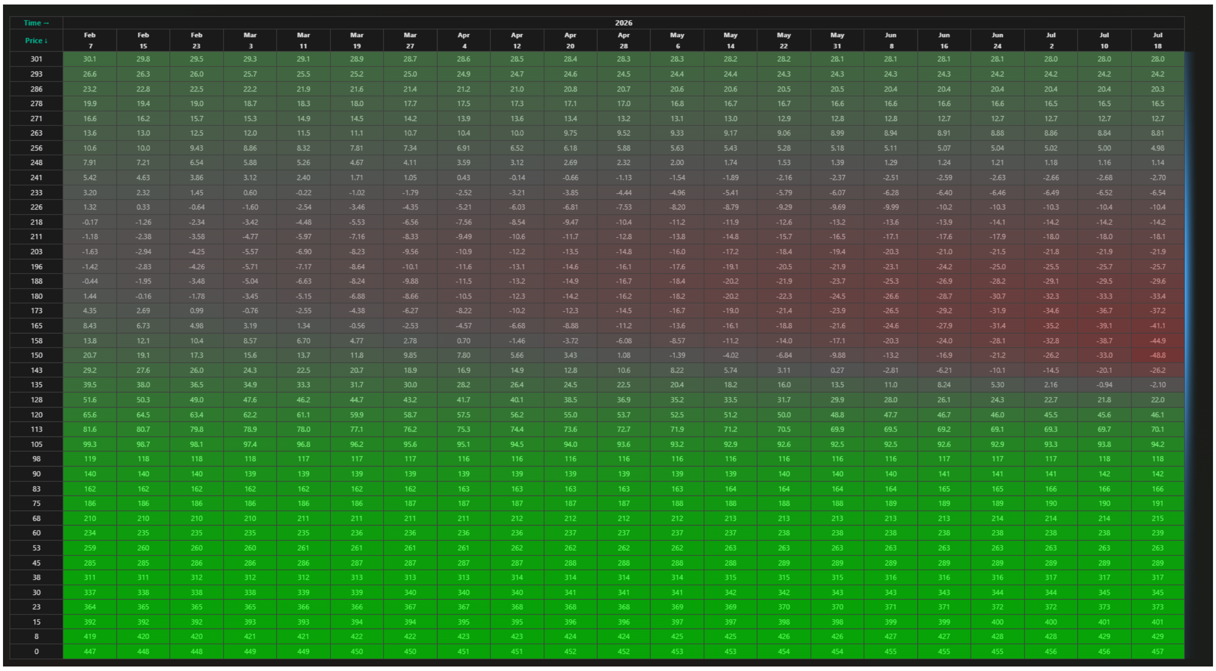
All results from Basic mode, plus:



Found Option Strategy card with legs, weights, expected return, and key statistics.



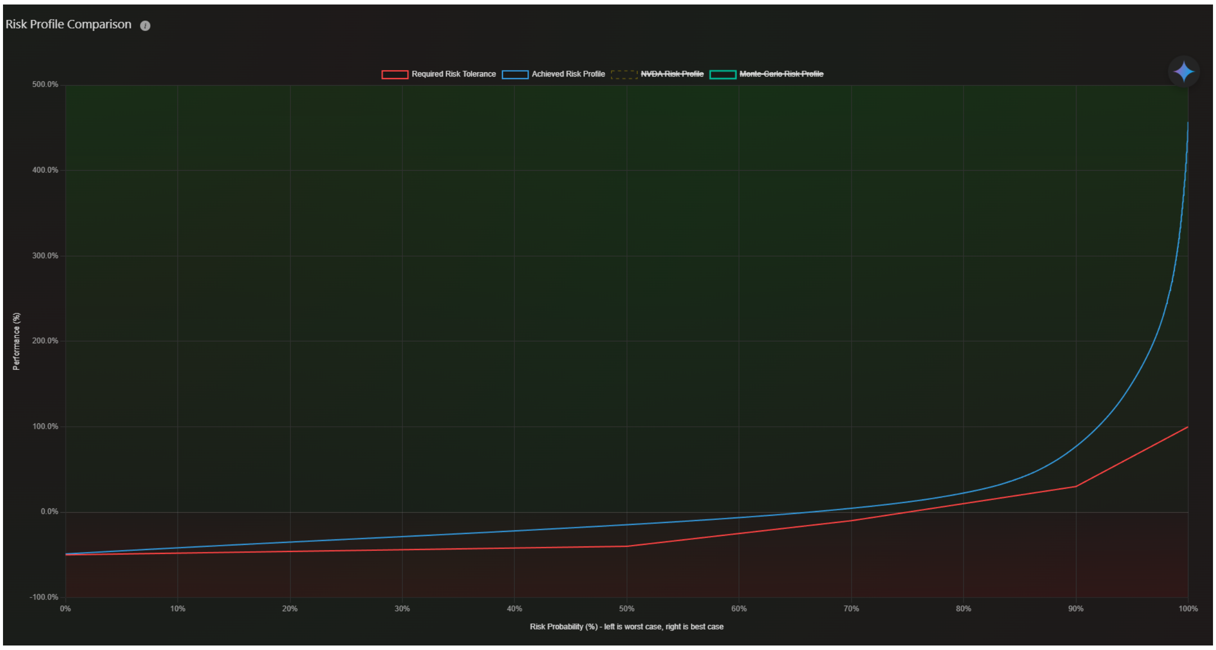
Strategy Performance Analysis: percentage return across all possible stock prices.



Time →	2026																				
Price ↓	Feb 7	Feb 15	Feb 23	Mar 3	Mar 11	Mar 19	Mar 27	Apr 4	Apr 12	Apr 20	Apr 28	May 6	May 14	May 22	May 31	Jun 8	Jun 16	Jun 24	Jul 2	Jul 10	Jul 18
301	30.1	29.8	29.5	29.3	29.1	28.9	28.7	28.6	28.5	28.4	28.3	28.3	28.2	28.2	28.1	28.1	28.1	28.1	28.0	28.0	28.0
293	26.6	26.3	26.0	25.7	25.5	25.2	25.0	24.9	24.7	24.6	24.5	24.4	24.4	24.3	24.3	24.3	24.3	24.2	24.2	24.2	24.2
286	23.2	22.8	22.5	22.2	21.9	21.6	21.4	21.2	21.0	20.8	20.7	20.6	20.6	20.5	20.5	20.4	20.4	20.4	20.4	20.4	20.3
278	19.9	19.4	19.0	18.7	18.3	18.0	17.7	17.5	17.3	17.1	17.0	16.8	16.7	16.7	16.6	16.6	16.6	16.6	16.5	16.5	16.5
271	16.6	16.2	15.7	15.3	14.9	14.5	14.2	13.9	13.6	13.4	13.2	13.1	13.0	12.9	12.8	12.8	12.7	12.7	12.7	12.7	12.7
263	13.6	13.0	12.5	12.0	11.5	11.1	10.7	10.4	10.0	9.75	9.52	9.33	9.17	9.06	8.99	8.94	8.91	8.88	8.86	8.84	8.81
256	10.6	10.0	9.43	8.86	8.32	7.81	7.34	6.91	6.52	6.18	5.88	5.63	5.43	5.28	5.18	5.11	5.07	5.04	5.02	5.00	4.98
248	7.91	7.21	6.54	5.88	5.26	4.67	4.11	3.59	3.12	2.69	2.32	2.00	1.74	1.53	1.39	1.29	1.24	1.21	1.18	1.16	1.14
241	5.42	4.63	3.86	3.12	2.40	1.71	1.05	0.43	-0.14	-0.66	-1.13	-1.54	-1.89	-2.16	-2.37	-2.51	-2.59	-2.63	-2.66	-2.68	-2.70
233	3.20	2.32	1.45	0.60	-0.22	-1.02	-1.79	-2.52	-3.21	-3.85	-4.44	-4.96	-5.41	-5.79	-6.07	-6.28	-6.40	-6.46	-6.49	-6.52	-6.54
226	1.32	0.33	-0.64	-1.60	-2.54	-3.46	-4.35	-5.21	-6.03	-6.81	-7.53	-8.20	-8.79	-9.29	-9.69	-9.99	-10.2	-10.3	-10.3	-10.4	-10.4
218	-0.17	-1.26	-2.34	-3.42	-4.48	-5.53	-6.56	-7.56	-8.54	-9.47	-10.4	-11.2	-11.9	-12.6	-13.2	-13.6	-13.9	-14.1	-14.2	-14.2	-14.2
211	-1.18	-2.38	-3.58	-4.77	-5.97	-7.16	-8.33	-9.49	-10.6	-11.7	-12.8	-13.8	-14.8	-15.7	-16.5	-17.1	-17.6	-17.9	-18.0	-18.0	-18.1
203	-1.63	-2.94	-4.25	-5.57	-6.90	-8.23	-9.56	-10.9	-12.2	-13.5	-14.8	-16.0	-17.2	-18.4	-19.4	-20.3	-21.0	-21.5	-21.8	-21.9	-21.9
196	-1.42	-2.83	-4.26	-5.71	-7.17	-8.64	-10.1	-11.6	-13.1	-14.6	-16.1	-17.6	-19.1	-20.5	-21.9	-23.1	-24.2	-25.0	-25.5	-25.7	-25.7
188	-0.44	-1.95	-3.48	-5.04	-6.63	-8.24	-9.88	-11.5	-13.2	-14.9	-16.7	-18.4	-20.2	-21.9	-23.7	-25.3	-26.9	-28.2	-29.1	-29.5	-29.6
180	1.44	-0.16	-1.78	-3.45	-5.15	-6.88	-8.66	-10.5	-12.3	-14.2	-16.2	-18.2	-20.2	-22.3	-24.5	-26.6	-28.7	-30.7	-32.3	-33.3	-33.4
173	4.35	2.69	0.99	-0.76	-2.55	-4.38	-6.27	-8.22	-10.2	-12.3	-14.5	-16.7	-19.0	-21.4	-23.9	-26.5	-29.2	-31.9	-34.6	-36.7	-37.2
165	8.43	6.73	4.98	3.19	1.34	-0.56	-2.53	-4.57	-6.68	-8.88	-11.2	-13.6	-16.1	-18.8	-21.6	-24.6	-27.9	-31.4	-35.2	-39.1	-41.1
158	13.8	12.1	10.4	8.57	6.70	4.77	2.78	0.70	-1.46	-3.72	-6.08	-8.57	-11.2	-14.0	-17.1	-20.3	-24.0	-28.1	-32.8	-38.7	-44.9
150	20.7	19.1	17.3	15.6	13.7	11.8	9.85	7.80	5.66	3.43	1.08	-1.39	-4.02	-6.84	-9.88	-13.2	-16.9	-21.2	-26.2	-33.0	-48.8
143	29.2	27.6	26.0	24.3	22.5	20.7	18.9	16.9	14.9	12.8	10.6	8.22	5.74	3.11	0.27	-2.81	-6.21	-10.1	-14.5	-20.1	-26.2
135	39.5	38.0	36.5	34.9	33.3	31.7	30.0	28.2	26.4	24.5	22.5	20.4	18.2	16.0	13.5	11.0	8.24	5.30	2.16	-0.94	-2.10
128	51.6	50.3	49.0	47.6	46.2	44.7	43.2	41.7	40.1	38.5	36.9	35.2	33.5	31.7	29.9	28.0	26.1	24.3	22.7	21.8	22.0
120	65.6	64.5	63.4	62.2	61.1	59.9	58.7	57.5	56.2	55.0	53.7	52.5	51.2	50.0	48.8	47.7	46.7	46.0	45.5	45.6	46.1
113	81.6	80.7	79.8	78.9	78.0	77.1	76.2	75.3	74.4	73.6	72.7	71.9	71.2	70.5	69.9	69.5	69.2	69.1	69.3	69.7	70.1
105	99.3	98.7	98.1	97.4	96.8	96.2	95.6	95.1	94.5	94.0	93.6	93.2	92.9	92.6	92.5	92.5	92.6	92.9	93.3	93.8	94.2
98	119	118	118	118	117	117	117	116	116	116	116	116	116	116	116	116	117	117	117	118	118
90	140	140	140	139	139	139	139	139	139	139	139	139	139	140	140	140	141	141	141	142	142
83	162	162	162	162	162	162	162	163	163	163	163	163	164	164	164	164	165	165	166	166	166
75	186	186	186	186	186	186	187	187	187	187	187	188	188	188	188	189	189	189	190	190	191
68	210	210	210	210	211	211	211	211	212	212	212	212	213	213	213	213	213	214	214	214	215
60	234	235	235	235	235	236	236	236	236	237	237	237	237	238	238	238	238	238	238	238	239
53	259	260	260	260	261	261	261	261	262	262	262	262	263	263	263	263	263	263	263	263	263
45	285	285	286	286	286	287	287	287	287	288	288	288	288	289	289	289	289	289	289	289	289
38	311	311	312	312	312	313	313	313	314	314	314	314	315	315	315	316	316	316	317	317	317
30	337	338	338	338	339	339	340	340	340	341	341	341	342	342	343	343	343	344	344	345	345
23	364	365	365	365	366	366	367	367	368	368	368	369	369	370	370	371	371	372	372	373	373
15	392	392	392	393	393	394	394	395	395	396	396	397	397	398	398	399	399	400	400	401	401
8	419	420	420	421	421	422	422	423	423	424	424	425	425	426	426	427	427	428	428	429	429
0	447	448	448	449	449	450	450	451	451	452	452	453	453	454	454	455	455	455	456	456	457

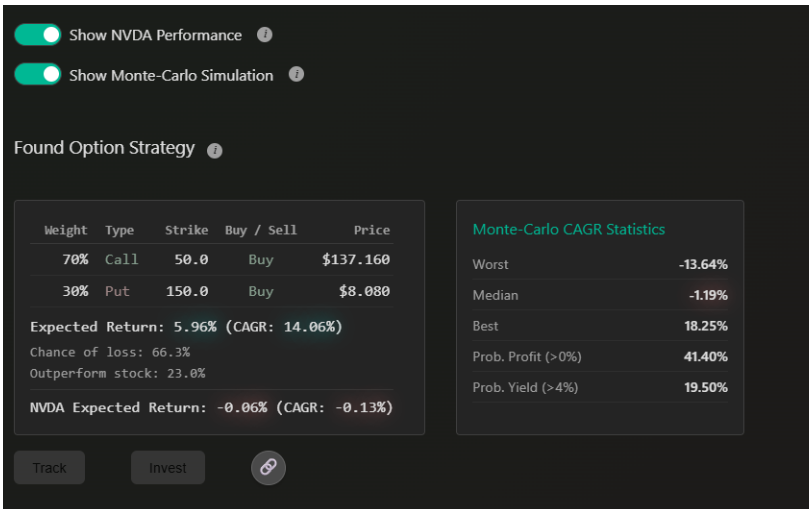
Black-Scholes table. The **blue glow on the right edge** represents the user's Assumed Probability Density: brighter glow indicates higher probability of the stock ending at that price.

- **Found Option Strategy:** The optimal spread with its legs, percentage weights, and expected return.
- **Risk Profile Comparison:** Overlay of the user's risk tolerance $\hat{r}_{tol}$ (red) and the strategy's actual risk profile $\hat{r}_c$ (blue).



The blue line (strategy) strictly dominates the red line (tolerance).

- **Underlying Performance Toggle:** "Show {ticker} Performance" reveals the stock's expected return and risk profile for comparison.
- **Outperform Probability:** The probability that the found strategy outperforms simply holding the stock, computed via spline intersections with the PDF.
- **Chance of Loss:** The probability that the strategy results in a net loss.
- **Monte-Carlo CAGR Statistics:** Simulated compound annual growth rate—see Risk Theory (Part IV) for why this matters beyond expected return.



Found strategy card with Monte-Carlo CAGR statistics.

8 Max Min Mode

Max Min takes a fundamentally different approach to portfolio construction. Instead of maximizing expected return (which depends on the accuracy of a user-supplied probability distribution), it maximizes the **worst-case outcome** (minimax optimization):

$$\max_{\mathbf{w}} \min_{S_T \in [0,\infty)} \sum_i w_i \, g_i(S_T)$$

where $\mathbf{w}$ is the weight vector over available option legs.

This is a **distribution-free** criterion: it makes no assumption about where the stock will end up. It answers the question: "If I had to prepare for the absolute worst, what is the strategy whose floor is the highest?"

8.1 Single Stock Mode

8.1.1 Workflow

1. **Enter a Ticker:** Same data pipeline as the other modes (price, options chain, implied PDFs).
2. **Select an Expiration Date:** Choose from the available expirations list.
3. **Launch Search:** The optimizer evaluates all spread combinations and finds the one whose minimum across all possible stock prices S_T is maximized.

8.1.2 Mathematical Formulation

For a single stock with available option payoff functions $\{g_i\}_{i\in[n]}$, the optimization problem is:

$$\max_{s\in\mathcal{S}} \min_{x\in\mathbb{R}} g_s(x) = \max_{\substack{o_i\in\mathcal{O} \\ w_i\in\mathbb{R} \\ \sum w_i=1}} \min_{x\in\mathbb{R}} \sum_i w_i \cdot g_{o_i}(x) \tag{9}$$

Because each g_i is piecewise linear (for vanilla options), the minimum over S_T occurs at a vertex of the piecewise-linear surface. The problem reduces to a finite-dimensional linear program.

8.2 Multi Stock Mode

Multi mode extends the minimax framework across multiple tickers. The user builds a portfolio by adding stocks one at a time, each with its own expiration and option chain.

8.2.1 Workflow

1. **Add Stocks:** Enter a ticker and click "Add." Repeat for each stock.
2. **Select Expirations:** Choose an expiration date per stock.
3. **Launch Search:** The optimizer finds both the cross-asset allocation *and* the per-stock option spread that maximizes the portfolio's worst case.

8.2.2 Cross-Asset Diversification

With M stocks, each having payoff functions $\{g_{m,i}\}$, the problem becomes:

$$\max_{\mathbf{w}_1,\ldots,\mathbf{w}_M,\, \boldsymbol{\alpha}\in\Delta^M} \min_{S_{T,1},\ldots,S_{T,M}} \sum_{m=1}^{M} \alpha_m \sum_i w_{m,i} \, g_{m,i}(S_{T,m})$$

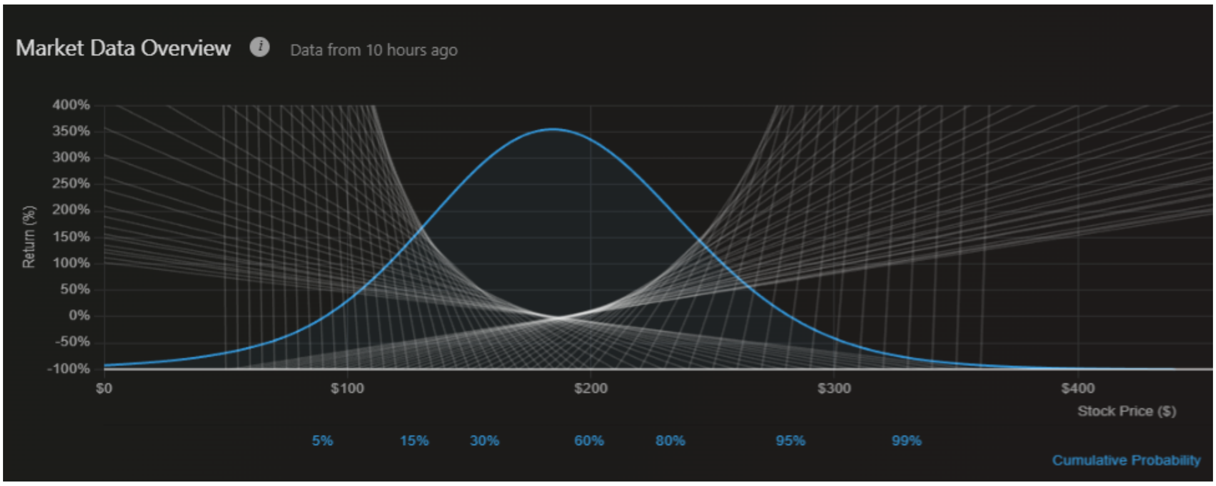
where $\boldsymbol{\alpha}$ is the inter-stock capital allocation and $\mathbf{w}_m$ is the intra-stock option spread. Because the stocks are independent, the global minimum decomposes into per-stock minima, and the outer allocation $\boldsymbol{\alpha}$ optimally diversifies across the per-stock floors.

Feature Isolation: Actions in Single mode (selecting tickers, dates) never affect the Multi mode state, and vice versa. This is enforced at the UI level.

Note: The Max Min optimization algorithm is currently in development. The UI for both sub-modes is complete and functional.

9 Market Data Overview

When a ticker and expiration are selected, the Market Data Overview chart is displayed:



Market Data Overview: thin white lines show individual option performance curves; the colored curve is the Implied PDF (peak $\sim$ \$180); the CDF is overlaid.

10 Implied Probability Distributions

10.1 What the Market "Thinks"

The Implied PDF is derived from current option prices using the following pipeline:

1. Compute implied volatility (IV) for each out-of-the-money option using the Black-Scholes formula.
2. Fit a **Quadratic Curve** to IV as a function of strike: $IV(K) = aK^2 + bK + c$.
3. Use the smoothed IV to generate dense Black-Scholes prices $C(K)$ across a fine strike grid.
4. Derive the PDF via the **Breeden-Litzenberger** finite-difference formula:

$$\mu(K) \approx e^{rT} \frac{C(K + \Delta K) - 2C(K) + C(K - \Delta K)}{(\Delta K)^2} \tag{10}$$

This **IV Smoothing** method avoids the instabilities (Runge's phenomenon) of fitting splines directly to noisy option prices.

11 Data Pipeline

- **Data Manager:** Serves market data to the frontend. Checks S3 data freshness and triggers updates if stale.

- **Data Updater:** Fetches live data from the Alpaca API when triggered.
- **Caching:** Three-layer architecture: S3 (backend), CloudFront (edge), Browser (30-second window).
- **Implied PDFs stay in sync:** Any options refresh automatically triggers a recalculation of all implied PDFs for that ticker.
- **Thundering Herd Protection:** Uses `isBeingUpdated` flag + Probabilistic Early Expiration (PEE) to prevent cache stampedes.

Part IV
Risk Theory

12 Risk Profiles

Every investment carries risk. CallCulator quantifies risk through two mathematical objects:

12.1 Personal Risk Tolerance Profile

A function $\hat{r}_{tol} : [0,1] \to \mathbb{R}$ mapping cumulative probability λ to the minimum acceptable return at that probability level.

Example: "I never want to lose more than 25% (at $\lambda = 0$), in 50% of cases I want to profit (at $\lambda = 0.5$, return ≥ 0), and my best case should be at least +100% (at $\lambda = 1$)."

12.2 Investment Risk Profile

For a strategy with performance function $p_c(x)$ and assumed PDF $\mu(x)$, the risk profile is:

$$\hat{r}_c(\lambda) = p_c\big(\Psi^{-1}(\lambda)\big), \qquad \Psi(x) := \int_0^x \mu(y)\,\mathrm{d}y \tag{11}$$

where Ψ is the cumulative distribution function. This maps each cumulative probability level $\lambda \in [0,1]$ to the corresponding return of the strategy.

12.3 Matching Profiles

A strategy is **valid** if its risk profile dominates the tolerance at every point:

$$\hat{r}_c(\lambda) \geq \hat{r}_{tol}(\lambda) \quad \forall\, \lambda \in [0,1]$$

The optimization engine searches for the valid strategy with the highest expected return.

13 On the Merit of Spreads

13.1 The Single-Step Argument Against Spreads

For a single, isolated investment, the expected return of a spread is:

$$\mathbb{E}\Big[\sum_i w_i\, g_i\Big] = \sum_i w_i\, \mathbb{E}[g_i] \tag{12}$$

Since this is a convex combination of real numbers, its maximum occurs at the boundary ($w_j = 1$ for some j). Therefore, in a single-step game, a single option always has the highest expected return. Spreads appear pointless.

13.2 The Multi-Step Counter-Argument: Geometric Growth

Investing is not a single event—it is a **repeated game**. In repeated games, maximizing the arithmetic expectation can lead to guaranteed ruin.

13.2.1 Alice and Bob: A Story of Ruin

Alice offers Bob a coin (80% Heads, 20% Tails):

- Heads: +100% (investment doubles)
- Tails: −100% (investment lost)

Rule: Bob must reinvest his entire bankroll.

Expected return per flip: +60%. But if Bob plays long enough:

$$P(\text{at least one Tails in } n \text{ flips}) = 1 - 0.8^n \xrightarrow{n\to\infty} 1$$

Bob is bankrupt with probability 1. The arithmetic expectation (+60%) is misleading; the **geometric growth rate** is $\ln(0) = -\infty$ on a Tails outcome.

13.2.2 Position Sizing Saves the Day

By betting only a fraction f of his capital, Bob's geometric growth rate per flip becomes:

$$G(f) = 0.8 \ln(1 + f) + 0.2 \ln(1 - f)$$

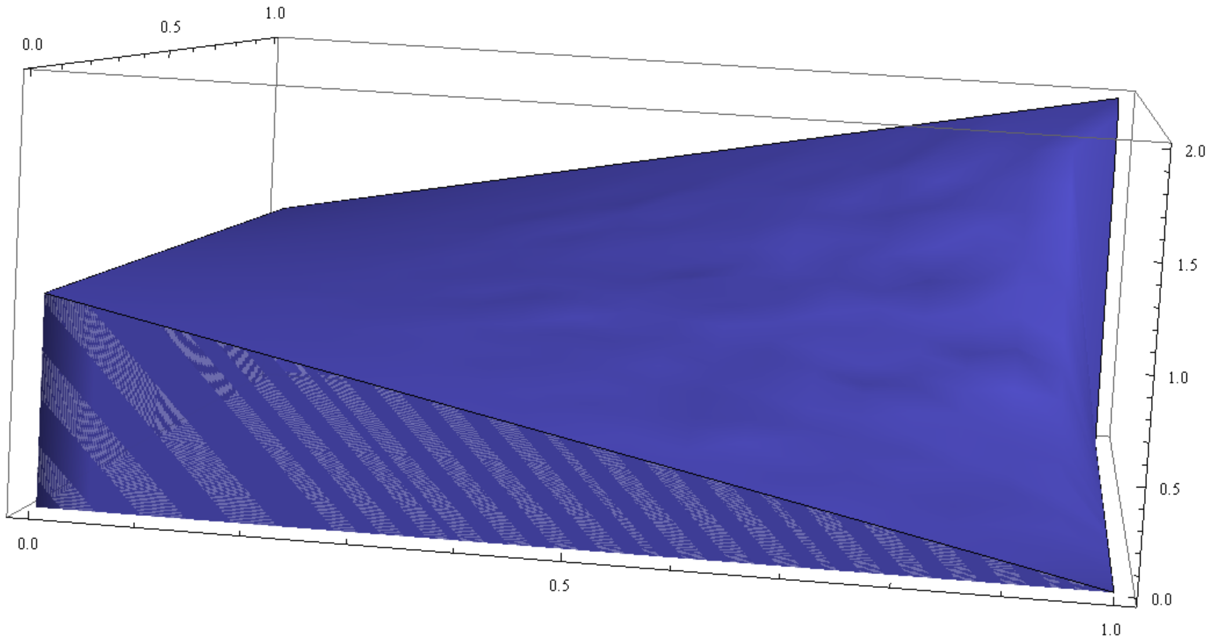
Maximizing $G(f)$ yields the **Kelly fraction**—the optimal bet size.

13.3 Beyond the Coin Flip: The Ergodicity Problem

This coin-flip example might seem extreme, but the core issue persists in almost all repeated investing. The mathematical term for this is **ergodicity breaking**.

In financial markets, the **Expected Return** (average of all possible outcomes) is often very different from the **Median Outcome** (what actually happens to most people). A strategy might have a positive Expected Return but still lead to a loss for 90% of investors over time because a few massive winners skew the average.

When you invest repeatedly (re-investing your gains), you don't experience the average of parallel universes; you experience one single path through time. If that path hits zero, you are out of the game. This is why maximizing Expected Return is not enough—you must manage the risk of the "time-average" diverging from the "ensemble-average."



Simulation of average return per flip vs. fraction of capital wagered.

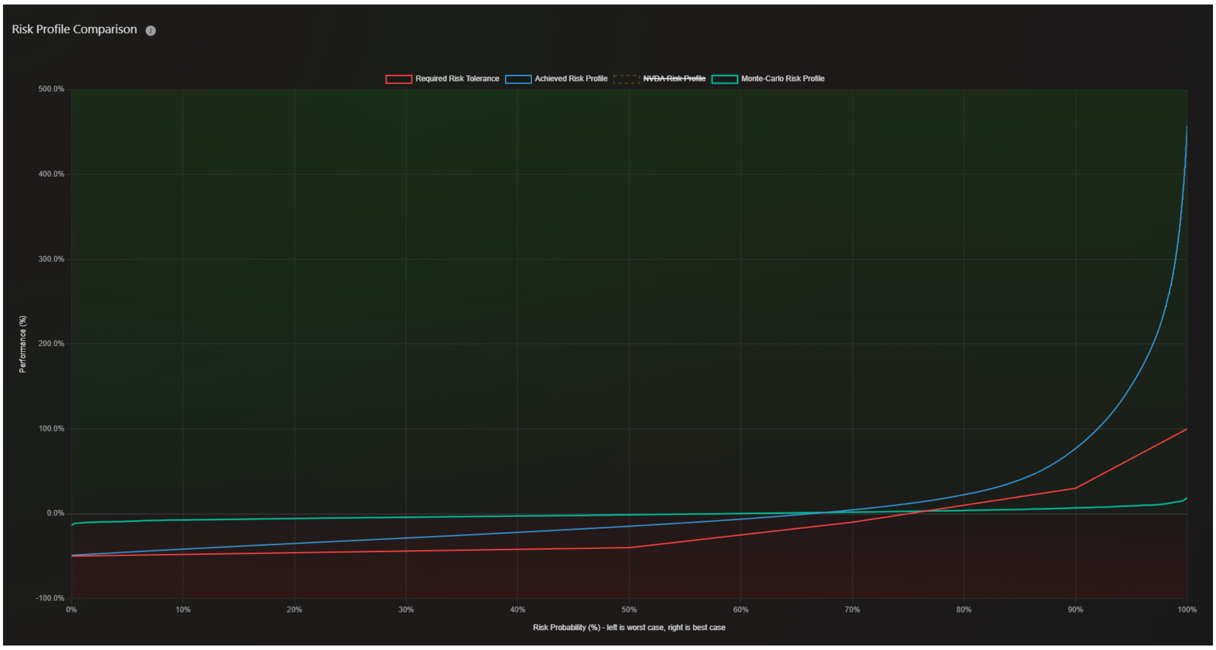
13.4 Monte-Carlo Simulation: Quantifying Hidden Risk

Expected return alone is an insufficient metric for investment decisions in a repeated-game context. Two strategies with identical $\mathbb{E}[g]$ can have vastly different geometric growth rates depending on the variance and tail behavior of their return distributions.

Monte-Carlo CAGR simulation exposes this:

1. Draw N samples $\{x_j\}_{j=1}^{N}$ from the user's PDF $\mu(x)$.
2. For each sample, compute the strategy return $g(x_j)$.
3. Simulate T rounds of compounding: $W_T = W_0 \prod_{t=1}^{T}(1 + g(x_{\sigma(t)}))$ where σ is a random draw.
4. Report the annualized geometric mean: $\text{CAGR} = (W_T/W_0)^{1/T} - 1$, averaged over many runs.

A high expected return with heavy tail risk yields a low (or negative) CAGR, revealing the ruin potential that a simple average conceals.



Risk Profile Comparison with Monte-Carlo insights.

13.5 Implications for Options

A spread is precisely this: instead of betting everything on a single call option (maximizing single-step expected return), a spread allocates capital across multiple payoff profiles, avoiding absorbing barriers (total loss) and maximizing long-term geometric growth. This is why Call-Culator's risk-constrained optimization produces spreads—they are the geometrically optimal strategy for repeated investing.

Part V
Reference

14 Quick Reference

- **Modes:** Basic Calculation, Probability & Risk, Max Min (tabs at top).
- **Show/Hide Math:** Toggle visibility of mathematical formulas. Setting persists across mode switches.
- **Data Freshness:** "Data from X ago" labels auto-update every minute.
- **Gemini AI:** Built-in chat assistant for questions about your analysis.
- **Share:** Generates a public URL; view counter increments per visit.
- **Auth & Billing:** Sign up via the header. Tier and credit balance are displayed. Session refreshes every 60 seconds.
- **Low-Performance Mode:** Automatically detected; reduces animations and shows a banner.

15 Glossary

- **PDF** – Probability Density Function. Describes the likelihood of each possible stock price.
- **CDF** – Cumulative Distribution Function. $\Psi(x) = \int_{-\infty}^{x} \mu(y)\, dy$.
- **IV** – Implied Volatility. The market's expectation of future volatility, back-solved from option prices via Black-Scholes.
- **Strike Price (K)** – The price at which an option can be exercised.
- **Premium (p)** – The market price of an option contract.
- **Spread** – A combination of two or more option legs.
- **CAGR** – Compound Annual Growth Rate. The annualized return assuming reinvestment.
- **Kelly Criterion** – The optimal fraction of capital to wager for maximum geometric growth.
- **PEE** – Probabilistic Early Expiration. A backend caching strategy that refreshes data before it strictly expires.